Assessment Schedule – 2006

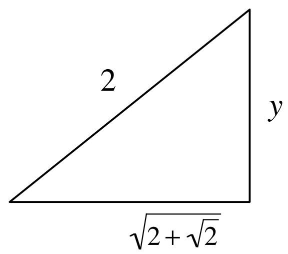
Scholarship Mathematics with Calculus (93202)

Evidence Statement

Question	Evidence	Code	Judgement
ONE (a)	The minute hand travels at $\frac{2\pi}{60} = \frac{\pi}{30}$ radians / minute Using the cosine rule with x = the distance between the tips of the hands, $x^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \times \cos\alpha$ $= 100 - 96\cos\alpha$ **EITHER** $2x\frac{\mathrm{d}x}{\mathrm{d}t} = 96\sin\alpha\frac{\mathrm{d}\alpha}{\mathrm{d}t}$ $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{48\sin\alpha}{\sqrt{100-96\cos\alpha}}\frac{\mathrm{d}\alpha}{\mathrm{d}t}$ **OR** $2x\frac{\mathrm{d}x}{\mathrm{d}\alpha} = 96\sin\alpha$ $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}\alpha} \bullet \frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{96\sin\alpha}{2x} \bullet \frac{\mathrm{d}\alpha}{\mathrm{d}t}$	I	
	The hour hand travels at $\frac{\frac{2\pi}{12}}{60} = \frac{\pi}{360}$ radians / minute If α is the angle between the hands then $\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \left(\frac{\pi}{30} - \frac{\pi}{360}\right) = \frac{11\pi}{360}$ radians / minute At 9am $\alpha = \frac{\pi}{2}$, $x = 10$ $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{48}{10} \times \frac{11\pi}{360}$ $= \frac{11\pi}{75}$ cm/min. The rate of change of the distance between the tips of the hands of the clock at 9 am is $\frac{11\pi}{75}$ cm/min.	N	Accept 0.46.

ONE (b)	**EITHER:** Given $\ln(1+x) \approx A + Bx + Cx^2$ and assuming equality, for all x, $-1 < x \le 1$ Putting $x = 0$ gives $A = 0$ Differentiating wrt x $\frac{1}{1+x} = B + 2Cx$ and putting $x = 0$, $B = 1$ Differentiating again wrt x $\frac{-1}{(1+x)^2} = 2C$ and putting $x = 0$, $C = -\frac{1}{2}$	**O** **N**	NB only $x = 0$ acceptable since it removes higher powers of x.
	OR: $\ln(1+x) \approx A + Bx + Cx^2$ $\frac{1}{1+x} = B + 2Cx$ and $\frac{-1}{(1+x^2)} = 2C$ but $(1+x)^{-1} = 1 - x + x^2 - x^3 + ...$ and equating coefficients $B = 1$, $2C = -1$, $C = -0.5$ So $\ln(1+x) \approx x - \frac{1}{2}x^2$ but when $x = 0$, $A = 0$. For $\left(1+\frac{1}{2n}\right)^{n+3} < \left(1+\frac{1}{n}\right)^{n-1}$, taking logs of both sides $(n+3)\ln\left(1+\frac{1}{2n}\right) < (n-1)\ln\left(1+\frac{1}{n}\right)$ and approximating from the above result $(n+3)\left(\frac{1}{2n} - \frac{1}{2(2n)^2}\right) < (n-1)\left(\frac{1}{n} - \frac{1}{2n^2}\right)$, $0 < \frac{1}{n} \le 1$, $n \ge 1$	**I**	I If calculated from this line by trial and error etc Award N if first result not proven
	$(n+3)\left(\frac{4n-1}{8n^2}\right) < (n-1)\left(\frac{2n-1}{2n^2}\right)$ and since $n > 0$ $(n+3)(4n-1) < 4(n-1)(2n-1)$ $8n^2 - 12n + 4 - 4n^2 - 11n + 3 > 0$ $4n^2 - 23n + 7 > 0$	**S**	
	$n > \frac{23 + \sqrt{529 - 112}}{8}$ since $n \ge 0$ $n > \frac{23 + 20.42}{8} = 5.428$, so the least integer is $n = 6$.		

ONE (c)	Vol, $V = \int_0^{\pi} \pi x^2 dy$ $= \pi \int_0^{\pi} \left(2a\cos(2y)+b\right)^2 \mathrm{d}y$ $= \pi \int_0^{\pi} \left(4a^2\cos^2(2y)+4ab\cos(2y)+b^2\right)\mathrm{d}y$	**S**	
	$= \pi \int_0^{\pi} \left(2a^2\cos 4y+2a^2+4ab\cos 2y+b^2\right)\mathrm{d}y$ $= \pi \left[\frac{1}{2}a^2\sin 4y+2ab\sin 2y+(2a^2+b^2)y\right]_0^{\pi}$	**N**	
	$= \pi\,((2\mathrm{a}^2+\mathrm{b}^2)\pi)$ $= \pi^2(2\mathrm{a}^2+\mathrm{b}^2)$	**I**	

TWO (a)	Using $\cos 2\theta = 2\cos^2\theta - 1$, $\cos\frac{\pi}{4} = 2\cos^2\frac{\pi}{8} - 1$ and $2\cos^2\frac{\pi}{8} = 1 + \frac{\sqrt{2}}{2} = \frac{2+\sqrt{2}}{2}$ so $\cos\frac{\pi}{8} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$.	**S**	
	$\frac{1+7i}{-3+4i} = \frac{(1+7i)(-3-4i)}{(-3+4i)(-3-4i)} = \frac{25-25i}{25} = 1-i$ **EITHER** Further $1 - i = \sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$ Solving $z^2 = \sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$, $z = \sqrt[4]{2}\operatorname{cis}\left(-\frac{\pi}{8}\right)$ or $\sqrt[4]{2}\operatorname{cis}\left(\frac{7\pi}{8}\right)$	**N**	Treat N and I lines independently Accept degrees and decimals here $\pm 1.0987 \pm 0.455i$ or equivalent I– for one root
	Either So hence since from (i) $\cos\frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$, and from $\sin\frac{\pi}{4} = 2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$ $\sin\frac{\pi}{8} = \frac{\frac{\sqrt{2}}{2}}{\sqrt{2+\sqrt{2}}} = \frac{\sqrt{2}}{2\left(\sqrt{2+\sqrt{2}}\right)}$  $y^2 = 4 - 2 - \sqrt{2} = 2 - \sqrt{2}$ $y = \sqrt{2-\sqrt{2}}$ $z = \sqrt[4]{2}\operatorname{cis}\left(-\frac{\pi}{8}\right) = \sqrt[4]{2}\left(\frac{\sqrt{2+\sqrt{2}}}{2} - i\frac{\sqrt{2-\sqrt{2}}}{2}\right) = \frac{\sqrt{2\sqrt{2}+2}}{2} - i\frac{\sqrt{2\sqrt{2}-2}}{2}$ or $\sqrt[4]{2}\operatorname{cis}\left(\frac{7\pi}{8}\right) = \sqrt[4]{2}\left(-\frac{\sqrt{2+\sqrt{2}}}{2} + i\frac{\sqrt{2-\sqrt{2}}}{2}\right) = -\frac{\sqrt{2\sqrt{2}+2}}{2} + i\frac{\sqrt{2\sqrt{2}-2}}{2}$ **Or**	**I**	Decimals NOT allowed here Or equivalent Or equivalent

	$z = \sqrt[4]{2}\,\text{cis}\left(-\frac{\pi}{8}\right) = \sqrt[4]{2}\left(\frac{\sqrt{2+\sqrt{2}}}{2} - \mathrm{i}\frac{\sqrt{2}}{2\left(\sqrt{2+\sqrt{2}}\right)}\right) = \frac{\sqrt{2\sqrt{2}+2}}{2} - \mathrm{i}\frac{\sqrt{2\sqrt{2}-2}}{2}$ or $\sqrt[4]{2}\,\text{cis}\left(\frac{7\pi}{8}\right) = \sqrt[4]{2}\left(-\frac{\sqrt{2+\sqrt{2}}}{2} + \mathrm{i}\frac{\sqrt{2}}{2\left(\sqrt{2+\sqrt{2}}\right)}\right) = -\frac{\sqrt{2\sqrt{2}+2}}{2} + \mathrm{i}\frac{\sqrt{2\sqrt{2}-2}}{2}$ **OR** Let $z = a + \mathrm{i}b$, so $z^2 = a^2 - b^2 + 2ab\mathrm{i}$ So $a^2 - b^2 = 1 \qquad 2ab = -1$. $a^2 - \left(\frac{1}{2a}\right)^2 = 1$ $4a^4 - 4a^2 - 1 = 0$ - $a^2 = \frac{4 \pm \sqrt{32}}{8} = \frac{1 \pm \sqrt{2}}{2}$ $a^2 = \frac{1+\sqrt{2}}{2} \quad (a^2 > 0)$ $a = \pm\sqrt{\frac{1+\sqrt{2}}{2}}$ and $b = -\frac{1}{2a} = \mp\frac{1}{2}\sqrt{\frac{2}{1+\sqrt{2}}} = \mp\frac{1}{2}\sqrt{\frac{2}{1+\sqrt{2}}}$ or $b^2 = a^2 - 1 \quad b^2 = \frac{1+\sqrt{2}}{2} - 1 = \frac{\sqrt{2}-1}{2}$ $b = \pm\sqrt{\frac{\sqrt{2}-1}{2}}$ so $z = \pm\sqrt{\frac{\sqrt{2}+1}{2}} \mp \mathrm{i}\sqrt{\frac{\sqrt{2}-1}{2}}$.	I	Or equivalent

TWO (b)

Either — **O**

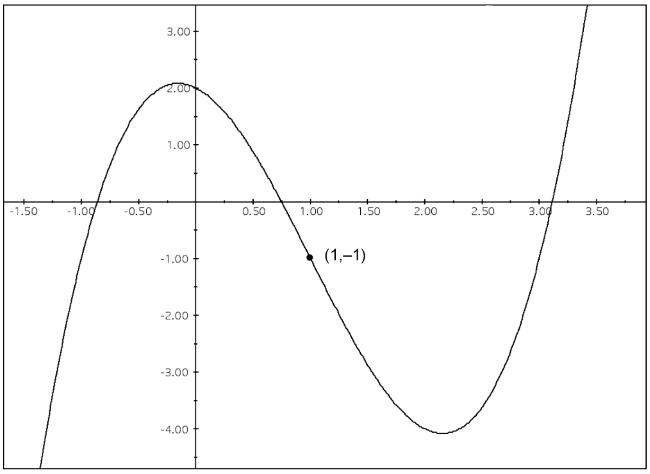
$f'(x) = 3x^2 - 6x - 1$
$f''(x) = 6x - 6$

So when $f''(x) = 0,\ x = 1$ and this is a point of inflection $\left(f'''(x) = 6 \neq 0\right)$ — **N**

$x = 1,\ f(x) = -1$ and so $(1, -1)$ is a point of inflection.

Since all cubics have rotational symmetry about their point of inflection we can write that — **I**

$g(x) = x^3 - 3x^2 - x + 2,\ x \geq 1$



OR

In general (a, b) is mapped to $(2–a, –b–2)$

Hence — **N**

$g(x) = -f(2 - x) - 2$

$= -(2 - x)^3 + 3(2 - x)^2 + (2 - x) - 2 - 2$ — **I**

$= x^3 - 6x^2 + 12x - 8 + 3x^2 - 12x + 12 - x - 2$

$= x^3 - 3x^2 - x + 2.$

OR

Translate f so that $(1, –1)$ goes to the origin

$f_1(x) = (x + 1)^3 - 3(x + 1)^2 - (x + 1) + 2 + 1$

Then reflect this function in each of the axes to get the rotation (or use $(x,y) \to (–x,–y)$) — **N**

ie $g_1(x) = -f_1(-x) = -(-x + 1)^3 + 3(-x + 1)^2 + (-x + 1) - 3$

And now translate back to $(1, –1)$

$g(x) = -\left(-(x - 1) + 1\right)^3 + 3\left(-(x - 1) + 1\right)^2 + \left(-(x - 1) + 1\right) - 3 - 1$

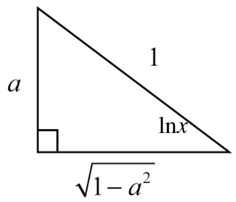
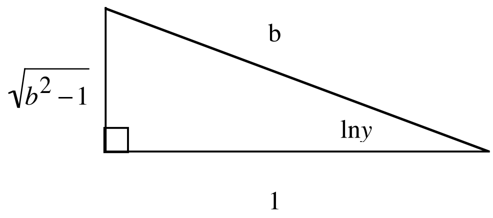
$g(x) = -(2 - x)^3 + 3(2 - x)^2 + (2 - x) - 4$ — **I**

$= x^3 - 6x^2 + 12x - 8 + 12 - 12x + 3x^2 + 2 - x - 4$

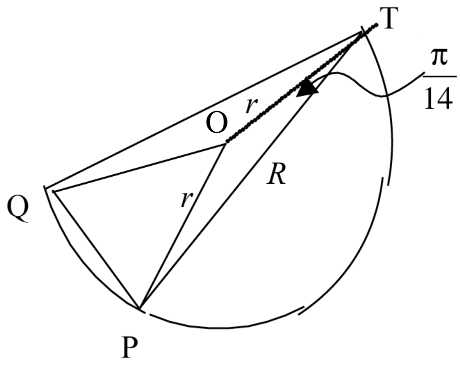
	$= x^3 - 3x^2 - x + 2$, $x \geq 1$ **OR** Let $g(x) = ax^3 + bx^2 + cx + d$ Use symmetry to find 4 points on the graph of $g(x)$. For example, $(1, -1)$ is on the graph of $g(x)$. $(0, 2)$ is on $f(x)$ so $(2, -4)$ is on $g(x)$ $(-1, -1)$ is on $f(x)$ so $(3, -1)$ is on $g(x)$ $(-2, -16)$ is on $f(x)$ so $(4, 14)$ is on $g(x)$ Substitute into $g(x) = ax^3 + bx^2 + cx + d$ and solve the 4 equations for *a, b, c* and *d* (this may require a graphic calculator) to get $a = 1$, $b = -3$, $c = -1$, and $d = 2$.	**N** **I**	$x \geq 1$ not required Any 2 correct points Any 4 correct points

THREE (a)	$y=\sin(\ln x), \quad x\ge 0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x}\cos(\ln x)$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{-1}{x^2}\sin(\ln x)-\frac{1}{x^2}\cos(\ln x)$ $=\frac{-y}{x^2}-\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}x}$ **EITHER** $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+x\frac{\mathrm{d}y}{\mathrm{d}x}+y=k$ $-y-x\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}y}{\mathrm{d}x}+y=k$ $\Rightarrow\ k=0$	S	
	OR $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+x\frac{\mathrm{d}y}{\mathrm{d}x}+y=k$ $x^2\left(\frac{-1}{x^2}\sin(\ln x)-\frac{1}{x^2}\cos(\ln x)\right)+x\left(\frac{1}{x}\cos(\ln x)\right)+\sin(\ln x)$ $=-\sin(\ln x)-\cos(\ln x)+\cos(\ln x)+\sin(\ln x)$ $=0$ $\Rightarrow\quad k=0$	I	
	When $\frac{\mathrm{d}\left(x^2\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{\mathrm{d}x}=x\frac{\mathrm{d}y}{\mathrm{d}x}-y+5$ $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x\frac{\mathrm{d}y}{\mathrm{d}x}=x\frac{\mathrm{d}y}{\mathrm{d}x}-y+5$ $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+x\frac{\mathrm{d}y}{\mathrm{d}x}+y=5$ So $k=5$ in the above. **EITHER** Hence try $y=\sin(\ln x)+5$ as a solution LHS $=x^2\left(\frac{-1}{x^2}\sin(\ln x)-\frac{1}{x^2}\cos(\ln x)\right)+x\left(\frac{1}{x}\cos(\ln x)\right)+\sin(\ln x)+5$ $=-\sin(\ln x)-\cos(\ln x)+\cos(\ln x)+\sin(\ln x)+5=5=$RHS So $y=\sin(\ln x)+5$ is a solution. **OR** Hence try $y=\sin(\ln x)+nx^2+mx+5$ as a solution	I	Accept $y=\sin(\ln x)+5$ if just stated. Or with $n=0$

	$x^2\left(\frac{-1}{x^2}\sin(\ln x)-\frac{1}{x^2}\cos(\ln x)+2n\right)+x\left(\frac{1}{x}\cos(\ln x)+2nx+m\right)$ $+\sin(\ln x)+n^2x+mx+5=5$ and $-\sin(\ln x)-\cos(\ln x)+2nx^2+\cos(\ln x)+2nx^2+$ $mx+\sin(\ln x)+nx^2+mx=0$ and $n=m=0$ So $y=\sin(\ln x)+5$ is a solution.		

THREE (b)	$\sin(\ln x) = a \qquad \cos(\ln y) = \frac{1}{b}$  $\tan(\ln(xy)).\tan\left(\ln\left(\frac{x}{y}\right)\right) = \tan(\ln x + \ln y).\tan(\ln x - \ln y)$ $= \frac{\tan(\ln x) + \tan(\ln y)}{1-\tan(\ln x).\tan(\ln y)}.\frac{\tan(\ln x) - \tan(\ln y)}{1+\tan(\ln x).\tan(\ln y)}$ $= \frac{\tan^2(\ln x) - \tan^2(\ln y)}{1-\tan^2(\ln x).\tan^2(\ln y)}$	**O**	
	$= \frac{\frac{a^2}{1-a^2} - \frac{b^2-1}{1}}{1 - \frac{a^2}{1-a^2}.\frac{b^2-1}{1}}$ $= \frac{a^2 - (b^2-1)(1-a^2)}{1-a^2-a^2(b^2-1)}$ $= \frac{a^2 - b^2 + a^2b^2 + 1 - a^2}{1-a^2-a^2b^2+a^2} = \frac{a^2b^2 - b^2 + 1}{1-a^2b^2}$	**I**	Or equivalent in sin and cos
	hence when $\frac{a^2b^2 - b^2 + 1}{1-a^2b^2} = 1$ $a^2b^2 - b^2 + 1 = 1 - a^2b^2$ $2a^2b^2 - b^2 = 0$ $b^2(2a^2-1) = 0$ $b^2 \neq 0 \quad a^2 = \frac{1}{2}$ $a = \pm\frac{1}{\sqrt{2}}$ **OR** $\tan(\ln(xy))\tan\left(\ln\left(\frac{x}{y}\right)\right) = 1$ $\sin(\ln(xy))\sin\left(\ln\left(\frac{x}{y}\right)\right) = \cos(\ln(xy))\cos\left(\ln\left(\frac{x}{y}\right)\right)$ $\cos\left(\ln(xy) + \ln\left(\frac{x}{y}\right)\right) = 0$	**S**	

	$\cos(2\ln(x)) = 0$ $1 - 2\sin^2(\ln(x)) = 0$ $1 - 2a^2 = 0$ $a = \pm\frac{1}{\sqrt{2}}$		

FOUR (a)	 The length of one arc of the coin is $R \cdot \frac{\pi}{7}$	**S** **N**	N and I points assessed separately
	Method 1 Let O be the centre of the coin. Triangle OPT is isosceles, so $R = 2r\cos\frac{\pi}{14}$ or $2r\sin\frac{3\pi}{7}$	**I**	Accept 1.949r
	Hence the ratio of the circumference of the 50p coin to that of the circle its vertices lie on is $\frac{\pi R}{2\pi r} = \frac{\pi 2r\cos\frac{\pi}{14}}{2\pi r} = \cos\frac{\pi}{14}$. **Method 2** Using the cosine rule (or sin rule) in triangle OPT $R^2 = 2r^2 - 2r^2\cos\frac{6\pi}{7} = 2r^2\left(1-\cos\frac{6\pi}{7}\right)$	**I**	Accept 0.975 Accept degrees
	Hence the ratio of the circumference of the 50p coin to that of the circle its vertices lie on is $\frac{\pi R}{2\pi r} = \frac{\pi\sqrt{2r^2\left(1-\cos\frac{6\pi}{7}\right)}}{2\pi r} = \frac{\sqrt{2\left(1-\cos\frac{6\pi}{7}\right)}}{2} = \frac{\sqrt{2\left(1-\left(1-2\sin^2\frac{3\pi}{7}\right)\right)}}{2}$ $= \sin\frac{3\pi}{7}$. **Note:** $\sin\frac{3\pi}{7} = \cos\left(\frac{\pi}{2}-\frac{3\pi}{7}\right) = \cos\frac{\pi}{14}$.		Accept $\frac{\sqrt{2\left(1-\cos\frac{6\pi}{7}\right)}}{2}$ Accept reciprocal (1.026 in decimals)

FOUR (b)

For family 1, with $y = f(x)$: $y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2y + x$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 + \dfrac{x}{y}$ $(= 2 + z)$ **O**

So for family 2, with $y = g(x)$: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{y}{2y + x}$ **N**

EITHER

$\dfrac{\mathrm{d}x}{\mathrm{d}y} = -\dfrac{2y + x}{y} \qquad y\dfrac{\mathrm{d}x}{\mathrm{d}y} + x = -2y$

Integrating with respect to y

$$\int\left(y\frac{\mathrm{d}x}{\mathrm{d}y} + x\right)dy = -\int 2y\,dy$$

S

$xy = -y^2 + c$

$y(x + y) = c$

OR

$(2y + x)\dfrac{\mathrm{d}y}{\mathrm{d}x} = -y \qquad x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = -2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$

Integrating with respect to x

$$\int\left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right)dx = -\int 2y\frac{\mathrm{d}y}{\mathrm{d}x}dx$$

S

$xy = -y^2 + c$

$y(x + y) = c$

OR

If we let $z = \dfrac{x}{y}$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{y}{2y + x} = -\dfrac{1}{2 + z}$

and $\dfrac{\mathrm{d}z}{\mathrm{d}x} = \dfrac{1}{y} - \dfrac{x}{y^2}\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{y}\left(1 + z\left(\dfrac{1}{2 + z}\right)\right)$

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{2z}{x}\left(\frac{1 + z}{2 + z}\right)$$

S

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and separating the variables and integrating

$$\int \frac{2 + z}{z(1 + z)}\,\mathrm{d}z = 2\int \frac{1}{x}\,\mathrm{d}x$$

using the given result

$$\int \frac{2}{z} - \frac{1}{z + 1}\,\mathrm{d}z = 2\int \frac{1}{x}\,\mathrm{d}x$$

$$2\ln z - \ln(z + 1) = 2\ln x + C$$

$$\ln\left(\frac{z^2}{z + 1}\right) = \ln\left(kx^2\right) \qquad \text{where } \ln k = C$$

	$\frac{z^2}{z+1} = kx^2$ $\frac{x^2}{y(x+y)} = kx^2$ $\frac{1}{y(x+y)} = k,\ x \neq 0$ $ky(x+y) = 1$ **OR** $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{y}{2y+x} = -\frac{1}{2+z}$ $z = \frac{x}{y}$ so $\frac{dz}{dy} = \frac{1}{y}\frac{dx}{dy} - \frac{x}{y^2} = \frac{dz}{dy} = -\frac{1}{y}(2+z) - \frac{z}{y} = -\frac{2(1+z)}{y}$ $\int \frac{1}{1+z}dz = -2\int \frac{1}{y}dy$ $\ln(1+z) = -2\ln(y) + \ln C$ $1+z = \frac{C}{y^2}$ $1+\frac{x}{y} = \frac{C}{y^2} \qquad y^2 + xy = C$ $y(x+y) = C$		

<table>
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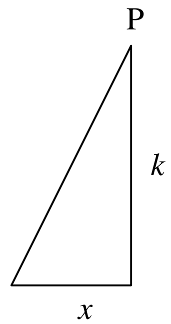
<td>FIVE
(a)</td>
<td>
Since the direction of motion is given by the gradient,
differentiating $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ wrt x we get
$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and for $P=\left(a\cos\theta, b\sin\theta\right)$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{b^2x}{a^2y}=-\frac{b\cos\theta}{a\sin\theta}=-\frac{b}{a}\cot\theta$
So the gradient of the normal at P is $\frac{a}{b}\tan\theta$
- -
EITHER
Equation of this normal is
$y-b\sin\theta=\frac{a}{b}\tan\theta(x-a\cos\theta)$
when $x=0$ and $\theta=\frac{\pi}{4}$
$y-b\frac{\sqrt{2}}{2}=\frac{a}{b}\left(-a\frac{\sqrt{2}}{2}\right)$ and $y=\frac{\sqrt{2}}{2b}\left(b^2-a^2\right)$.
- -
So the vertical distance below the level of P is
$b\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2b}\left(b^2-a^2\right)=\frac{\sqrt{2}}{2b}\left(a^2-b^2+b^2\right)=\frac{\sqrt{2}a^2}{2b}$.

OR
When $\theta=\pi/4$
Gradient of the normal at P is $\frac{a}{b}$, and $P=\left(\frac{a\sqrt{2}}{2},\frac{a\sqrt{2}}{2}\right)$
The distance required, say k, is given by
$\frac{k}{\left(\frac{a\sqrt{2}}{2}\right)}=\frac{a}{b}$ and $k=\frac{a}{b}.\left(\frac{a\sqrt{2}}{2}\right)=\frac{\sqrt{2}a^2}{2b}$.


</td>
<td>S

N

I</td>
<td>Early substitution of $\theta=\pi/4$ good</td>
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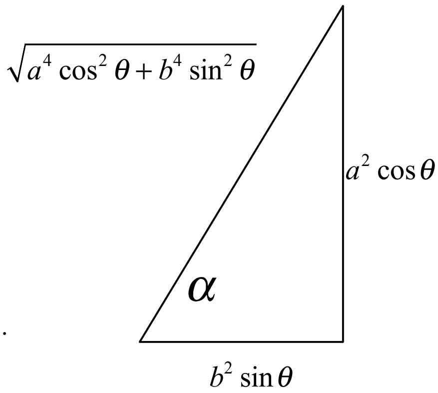
FIVE (b)	Let Q = $\left(a\cos\alpha, b\sin\alpha\right)$ Gradient of OP = $\dfrac{b\sin\theta}{a\cos\theta} = \dfrac{b}{a}\tan\theta$ Gradient of OQ = $\dfrac{b}{a}\tan\alpha$ So $\left(\dfrac{b}{a}\tan\theta\right)\left(\dfrac{b}{a}\tan\alpha\right) = -1$	O	
	$\tan\theta\tan\alpha = -\dfrac{a^2}{b^2}$ and $\tan\alpha = -\dfrac{a^2}{b^2}\cot\theta$ Hence $\sin\alpha = \dfrac{a^2\cos\theta}{\sqrt{a^4\cos^2\theta + b^4\sin^2\theta}}$ and $\cos\alpha = -\dfrac{b^2\sin\theta}{\sqrt{a^4\cos^2\theta + b^4\sin^2\theta}}$ So $Q = \left(-\dfrac{ab^2\sin\theta}{\sqrt{a^4\cos^2\theta + b^4\sin^2\theta}}, \dfrac{ba^2\cos\theta}{\sqrt{a^4\cos^2\theta + b^4\sin^2\theta}}\right)$.  	I	
	Method 2 Let Q be $\left(x_1, y_1\right)$ Gradient of OP = $\dfrac{b\sin\theta}{a\cos\theta} = \dfrac{b}{a}\tan\theta$ Gradient of OQ $= \dfrac{y_1}{x_1} = \dfrac{-a}{b}\cot\theta = \dfrac{-a\cos\theta}{b\sin\theta}$ $y_1 = \dfrac{-ax_1\cos\theta}{b\sin\theta}$	S	To include diagram or equivalent
	$\dfrac{x_1^2}{a^2} + \dfrac{y_1^2}{b^2} = 1$ $y_1^2 = b^2\left(1 - \dfrac{x_1^2}{a^2}\right)$ $= \dfrac{b^2}{a^2}\left(a^2 - x_1^2\right)$ $\dfrac{a^2x_1^2\cos^2\theta}{b^2\sin^2\theta} = \dfrac{b^2}{a^2}\left(a^2 - x_1^2\right)$	I	
	$x_1^2\left(\dfrac{a^2\cos^2\theta}{b^2\sin^2\theta} + \dfrac{b^2}{a^2}\right) = b^2$ $x_1^2 = \dfrac{a^2b^4\sin^2\theta}{a^4\cos^2\theta + b^4\sin^2\theta}$ $x_1 = \dfrac{\pm ab^2\sin\theta}{\sqrt{a^4\cos^2\theta + b^4\sin^2\theta}}$ $y_1 = \dfrac{\mp a^2b\cos\theta}{\sqrt{a^4\cos^2\theta + b^4\sin^2\theta}}$	S	

	So $Q = \left(\dfrac{\pm ab^2 \sin\theta}{\sqrt{a^4\cos^2\theta + b^4\sin^2\theta}}, \dfrac{\mp a^2 b\cos\theta}{\sqrt{a^4\cos^2\theta + b^4\sin^2\theta}} \right)$ Oneß equivalent solution is $Q = \left(\dfrac{\pm ab^2}{\sqrt{a^4\cot^2\theta + b^4}}, \dfrac{\mp a^2 b\cot\theta}{\sqrt{a^4\cot^2\theta + b^4}} \right)$		Accept $\pm$ or $\mp$ Or equivalent